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Study of orbit stabilizer theorem on Group structure of K1,3 and Diamond graph

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Abstract

Graph theory is the branch of mathematics that deals with points and lines connecting them. Group Theory is the branch of mathematics that studies the set and its properties with a particular binary operation defined on it. The researchers have experimented and used their imagination to explore these two branches of Mathematics. It has also helped researchers to do some creative thinking of their own. Graphs can be used to the model situation that occurs within certain kinds of problems. Using the group of automorphism and orbit-stabilizer Theorem, a link is established between Graph theory and Group theory. This paper aims to study the group structure of the automorphism group of a bipartite graph $K_{1,3}$ and Diamond graph. Further, the orbit-stabilizer theorem is used to study the structural properties of an automorphism group of $K_{1,3}$ and Diamond graphs.

Keywords: *Graph Theory, Group Theory, Automorphism, Bipartite graph $K_{m,n}$, Symmetric group S_n, Klein's 4-group V_4, orbit of a point, stabilizer of a point.*

Introduction

Let G (V, E) be a graph where V is the set of all vertices and E is the set of all edges. Define identity map f: G (V, E) → G (V, E) as $f(v) = v$ for $v \in V$. The function α: V(G)→V(G) is called automorphism if α maps a vertex of the same degree to a vertex of same degree [1,2].

Arthur Cayley states that every group is isomorphic to a group of permutations [1,6]. The automorphism group of a graph is a group of permutations. The group of all permutations on a set of cardinality n is called the symmetric group S_n and its order is n! The automorphism group of a graph G of order n is a subgroup of S_n [1,6].

This paper aims to explore the relationship between Graphs and Groups. The concept of the automorphism group of a graph is the main peripheral [6]. The binary operation is a composition of a permutation group, which aid in computing automorphism groups of graphs.

The content of this paper is a broad discussion about graphs $K_{1,3}$, the Diamond graph whose Automorphism group is a subgroup of S_n and Klein 4-group V_4 respectively [1]. The orbit-stabilizer theorem is applied to study the structural properties of the automorphism group of $K_{1,3}$ and diamond graph.

Basic Notations and terminology of Graph Theory

Some of the basic graph theory notations and terminologies, for example, a graph G (V, E), Automorphism group of a graph Aut(G), Degree of a vertex, Orbit of a graph, stabilizer of an element are taken from the book [1,6].

Some of the basic group theory notations and terminologies, for example, identity map, permutation, the composition of permutations, isomorphism, automorphism, the automorphism group of G, inverse of the element, Group table, Klein 4-group V_4, order of Group, symmetric group S_n of order n, orbit-stabilizer Theorem are taken from the book [1,4,5,7].

1. Types of Graphs

There are many types of graphs like paths, cycles, complete graphs, regular graph wheel, etc. the graphs which are presented in the paper are bipartite and diamond graphs. In this paper, the focus is on $K_{1,3}$ the Bipartite graph (star graph), and the diamond graph.

Bipartite graph G

A graph G is said to be bipartite if vertex set V(G) can be partitioned into two non-empty subsets U and W so that every edge of G has one vertex in U and other in W. [6]

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$K_{1,3}$ is a Bipartite graph (star graph), as every edge of **$K_{1,3}$** joins a vertex of U = {x} and a vertex of W = {u, v, w}. [1,2]

Diamond graph

The diamond graph is a planar undirected graph with 4-vertices and 5-edges. It consists of a complete graph minus one edge. [1,7]

2. Some group related definitions and theorem for graph.

1. The orbit of a graph: For a vertex v of a graph G, the set of all vertices into which v can be mapped by some automorphism of G is called an orbit of G [1,2].

2. The automorphism group of a graph G: An automorphism of a graph G is an isomorphism of G with itself. The set of all automorphisms of G forms a permutation group, Aut(G), acting on the vertex set V(G). This Aut(G) is called the automorphism group of G.

The automorphism group of G.

3. The orbit of a point: Let G be a group of permutations of a set S. For each s in S,
$orbit_G(s) = \{\sigma(s): \sigma \in G\}$ is subset of S called the orbit of s under G. [5,7]

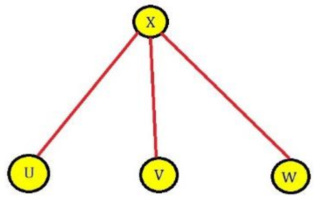
4. Stabilizer of a point: Let G be a group of permutations of a set S. for each I in S, let $stab_G(i)=\{\sigma \in G: \sigma(i) = i\}$.[5,7]

5. Orbit-Stabilizer Theorem: Let G be a finite group of permutation of a set S. Then for any i ∈ S, $|G| = |stab_G(i)|.|orbit_G(i)|$. [5,7]

3. Bipartite graph $K_{1,3}$ [1,3]:

Consider the graph **H = $K_{1,3}$**. The vertices of H are labelled as shown below.

H: labelled diagram of $K_{1,3}$



Consider automorphisms α_i: V(G)→ V(G), $1 \le i \le 6$ by

α_1= (x)(u)(v)(w) = e = identity, α_2 = (u, v, w), α_3 = (u, w,v), α_4 = (v, w), α_5 = (u, w),
α_6 = (u, v), where every automorphism of a graph is a permutation on V(G), automorphism can be expressed more simply, in terms of permutations cycles.

o	$\boldsymbol{\alpha_1}$	$\boldsymbol{\alpha_2}$	$\boldsymbol{\alpha_3}$	$\boldsymbol{\alpha_4}$	$\boldsymbol{\alpha_5}$	$\boldsymbol{\alpha_6}$
$\boldsymbol{\alpha_1}$	α_1	α_2	α_3	α_4	α_5	α_6
$\boldsymbol{\alpha_2}$	α_2	α_3	α_1	α_6	α_4	α_5
$\boldsymbol{\alpha_3}$	α_3	α_1	α_2	α_5	α_6	α_4
$\boldsymbol{\alpha_4}$	α_4	α_5	α_6	α_1	α_2	α_3
$\boldsymbol{\alpha_5}$	α_5	α_6	α_4	α_3	α_1	α_2
$\boldsymbol{\alpha_6}$	α_6	α_4	α_5	α_2	α_3	α_1

Table 1. Automorphism of $K_{1,3}$

From Table1,

$\alpha_1 o \alpha_1 = \alpha_1, \alpha_1 o \alpha_2 = \alpha_2, \alpha_1 o \alpha_3 = \alpha_3, \alpha_1 o \alpha_4 = \alpha_4, \alpha_1 o \alpha_5 = \alpha_5, \alpha_1 o \alpha_6 = \alpha_6$

implies $\alpha_1$is identity element of Aut(H).

$\alpha_2 o \alpha_3 = \alpha_1 = \alpha_3 o \alpha_2$ implies α_2 is a inverse of α_3 and conversely.

$\alpha_4 o \alpha_4 = \alpha_1$, $\alpha_5 o \alpha_5 = \alpha_1$, $\alpha_6 o \alpha_6 = \alpha_1$ implies $\alpha_4, \alpha_5, \alpha_6$ are their own inverse. Multiplication of any two elements of Aut(H) is again an element of Aut(H).

Therefore, **Aut(H) is closed under multiplication** (composition)

Set of these permutations form a non-abelian group under composition called automorphism group of H denoted by Aut(H).

Aut(H) = $\{\boldsymbol{\alpha_1 = e(identity), \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6}\}$ and $|Aut(H)| = 6$

such that $\boldsymbol{\alpha_2 \alpha_3 = e, \alpha_6{}^2 = \alpha_4{}^2 = \alpha_5{}^2 = e}$ **is group of order 6 isomorphic to S_3.**

$orbit_{Aut(H)}(u) = \{u, v, w\}$,

$orbit_{Aut(H)}(v) = \{u, v, w\}$,

$orbit_{Aut(H)}(w) = \{u, v, w\}$,

$orbit_{Aut(H)}(x) = \{x\}$

$|orbit_{Aut(H)}(u)| = 3$, $|orbit_{Aut(H)}(v)| = 3$, $|orbit_{Aut(H)}(w)| = 3$, $|orbit_{Aut(H)}(x)| = 1$

Orbits of graph H are {x}, {u,v,w}.

$\text{Stab}_{Aut(H)}(x) = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4 , \alpha_5 , \alpha_6\}$

$\text{Stab}_{Aut(H)}(u) = \{\alpha_1, \alpha_4\}$,

$\text{Stab}_{Aut(H)}(v) = \{\alpha_1, \alpha_5\}$,

$\text{Stab}_{Aut(H)}(w) = \{\alpha_1, \alpha_{6,}\}$

$|stab_{Aut(H)}(u)| = 2$, $|stab_{Aut(H)}(v)| = 2$, $|stab_{Aut(H)}(w)| = 2$, $|stab_{Aut(H)}(x)| = 6$

By the orbit-stabilizer Theorem

$|Aut(H)| = |orbit_{Aut(H)}(x)| \cdot |stab_{Aut(H)}(x)| = 1 \text{x } 6 = 6$

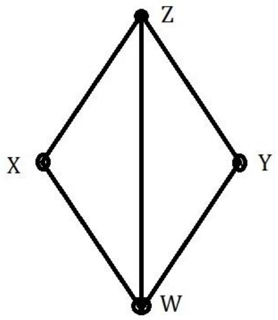
$|Aut(H)| = |orbit_{Aut(H)}(u)| \cdot |stab_{Aut(H)}(u)| = 3 \text{ x } 2 = 6$

$|Aut(H)| = |orbit_{Aut(H)}(w)| \cdot |stab_{Aut(H)}(w)| = 3 \text{ x } 2 = 6$

$|Aut(H)| = |orbit_{Aut(H)}(v)| \cdot |stab_{Aut(H)}(v)| = 3 \text{ x } 2 = 6$

4. Diamond graph [2,3]:

Consider the diamond graph F as follows:



Consider Automorphisms β_i: V(F)→ V(F), 1≤ $i \leq 4$ by

$\beta_1(v) = v$, ∀ vertices v of F, $\beta_2(v) = \begin{cases} x \text{ if } v = y \\ y \text{ if } v = x \end{cases}$,

$\beta_3(v) = \begin{cases} w \text{ if } v = z \\ z \text{ if } v = w \end{cases}$, $\beta_4(v) = \begin{cases} y \text{ if } v = x \\ x \text{ if } v = y \\ w \text{ if } v = z \\ z \text{ if } v = w \end{cases}$,

β_1=identity= e, $\beta_2 = (x, y)$, $\beta_3 = (z, w)$,$\beta_4 = (x, y)(z, w)$, where every automorphism of a graph is a permutation on V(G), automorphism can be expressed more simply, in terms of permutations cycles. The group table for Aut(F)

o	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$	$\boldsymbol{\beta_4}$
$\boldsymbol{\beta_1}$	β_1	β_2	β_3	β_4
$\boldsymbol{\beta_2}$	β_2	β_1	β_4	β_3
$\boldsymbol{\beta_3}$	β_3	β_4	β_1	β_2
$\boldsymbol{\beta_4}$	β_4	β_3	β_2	β_1

Table2: Automorphisms of Diamond graph

From Table2, $\beta_1 o \beta_1 = \beta_1, \beta_1 o \beta_2 = \beta_2, \beta_1 o \beta_3 = \beta_3, \beta_1 o \beta_4 = \beta_4, \beta_1 o \beta_5 = \beta_5, \beta_1 o \beta_6 = \beta_6$

implies $\beta_1$is identity element of Aut(F).

$\beta_2 o \beta_2 = \beta_1$, $\beta_3 o \beta_3 = \beta_1$, $\beta_4 o \beta_4 = \beta_1$ i.e $\beta_2, \beta_3, \beta_4$ are their own inverse. Multiplication of any two elements of Aut(F) is again a element of Aut(F). Therefore Aut(F) is closed under multiplication (composition)

Set of these permutations form a group under composition called automorphism group of F denoted by Aut(F).

Aut(F) = $\{\boldsymbol{\beta_1 = e(identity), \beta_2, \beta_3, \beta_4}\}$ such that $\boldsymbol{\beta_2}^2 = \boldsymbol{\beta_3}^2 = \boldsymbol{\beta_4}^2 = \boldsymbol{e}$ is abelian group of order 4 isomorphic to Klein 4-group V_4.

$orbit_{Aut(F)}(x) = \{x, y\}$,

$orbit_{Aut(F)}(y) = \{x, y\}$,

$orbit_{Aut(F)}(w) = \{z, w\}$

$orbit_{Aut(F)}(z) = \{z, w\}$

$|orbit_{Aut(F)}(x)| = 2$, $|orbit_{Aut(F)}(y)| = 2$, $|orbit_{Aut(F)}(w)| = 2$, $|orbit_{Aut(F)}(z)| = 2$

Orbits of graph F are {x,y},{z,w}.

$\text{Stab}_{Aut(F)}(x) = \{\beta_1, \beta_3\}$, $\text{Stab}_{Aut(F)}(y) = \{\beta_1, \beta_3\}$, $\text{Stab}_{Aut(F)}(w) = \{\beta_1, \beta_2\}$,

$\text{Stab}_{Aut(F)}(z) = \{\beta_1, \beta_2\}$

$|stab_{Aut(F)}(x)| = 2$, $|stab_{Aut(F)}(y)| = 2$, $|stab_{Aut(F)}(w)| = 2$, $|stab_{Aut(F)}(z)| = 2$

$|Aut(F)| = 4$

By the orbit-stabilizer Theorem

$|Aut(F)| = |orbit_{Aut(F)}(x)| \cdot |stab_{Aut(F)}(x)|$

$|Aut(F)| = |orbit_{Aut(F)}(y)| \cdot |stab_{Aut(F)}(y)|$

$|Aut(F)| = |orbit_{Aut(F)}(w)| \cdot |stab_{Aut(F)}(w)|$

$|Aut(F)| = |orbit_{Aut(F)}(z)| \cdot |stab_{Aut(F)}(z)|$

Conclusion:

Automorphism group for $K_{1,3}$ and Diamond graph is the symmetric group S_3 and Klein's 4-group V_4 respectively. A group is associated with each graph, knowns as the Automorphism group of a graph. Different groups can be associated with automorphisms of a graph and vice-versa. The orbit-stabilizer theorem can be verified for the given graphs. So, the results of these group structure apply to study the more complex graphs and their properties to develop interdisciplinary approach between groups and graphs.

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Conflicts of interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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