InSight Bulletin: A Multidisciplinary Interlink International Research Journal
Peer Reviewed International, Open Access Journal.
ISSN : 3065-7857 / Website: https://ibrj.us / Volume-3, Issue-1 / January - 2026

Original Article

Bianchi Type-IX Accelerating Universe with Modified Chaplygin Gas in Hoyle-Narlikar C-field Cosmology

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Manuscript ID:
IBMIIRJ -2026-030152

Submitted: 21 Dec. 2025

Revised: 29 Dec. 2025

Accepted: 16 Jan. 2026

Published: 31 Jan. 2026

ISSN: 3065-7857

Volume-3

Issue-1

Pp. 268-273

January 2026

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OPEN ACCESS

Quick Response Code:

Web. https://ibrj.us

DOI: 10.5281/zenodo.19182080

DOI Link:
https://doi.org/10.5281/zenodo.19182080

Abstract

This research investigates the dynamics of an anisotropic Bianchi type-IX spacetime incorporating a Modified Chaplygin Gas (MCG) within the context of the Hoyle-Narlikar creation field (C-field) framework. By utilizing the MCG equation of state, the model successfully bridges the early radiation phase and the late-time dark energy epoch. To achieve exact solutions for the gravitational field equations, we apply two physical constraints: a constant deceleration parameter (q) and a direct proportionality between the shear and expansion scalars. Our derived mathematical expressions for pressure, energy density, and scale factors demonstrate that the continuous generation of matter—driven by the negative energy of the C-field—prevents the universe from reaching a zero-density vacuum state at late times. Instead, the density stabilizes at a definitive constant. Furthermore, the analysis confirms an initial Big Bang singularity, followed by an accelerated expansion phase ($-1<q<0$), while preserving late-time anisotropy.

Keywords: *Bianchi Type-IX Space-time, Modified Chaplygin Gas C-field, Dark Energy*

Introduction

Studying different shapes of space and time is important for understanding how the universe started. Even though today's big-picture observations show the universe is the same in all directions, some strange patterns suggest it might have been very uneven in the early times. Because of this, the Bianchi Type-IX model is being looked at a lot. It can handle different movements and changes in space, and it can become the usual Friedmann-Robertson-Walker (FRW) model under certain conditions. The traditional Big Bang theory faces well-documented theoretical challenges, particularly the initial singularity and the horizon problem. To overcome these, Hoyle and Narlikar introduced a negative-energy, massless scalar field, known as the Creation Field (C-field), which facilitates ongoing matter creation. This theoretical framework was further developed by Narlikar and Padmanabhan to resolve the cosmological flatness problem. Recent studies, like those by Patil et al., looked at Bianchi Type-IX geometry with an ideal fluid and the C-field.

How to cite this article:
Mahajan, N., Swami, N., & Patil, V. R. (2026). Bianchi Type-IX Accelerating Universe with Modified Chaplygin Gas in Hoyle-Narlikar C-field Cosmology. InSight Bulletin: A Multidisciplinary Interlink International Research Journal, 3(1), 268–273. https://doi.org/10.5281/zenodo.19182080

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But their work was limited to a scenario where there was no pressure, meaning it was only dust-like matter (p=0). Similarly, Bali and Saraf studied Bianchi Type-I models in this context of the creation field. This paper takes things further by adding the Modified Chaplygin Gas (MCG) to the Bianchi Type-IX model. The MCG is a strong possibility for dark energy, and it follows a special equation that connects pressure and density (p=Aρ-B/ρ^α). Instead of using simple geometric rules, our approach uses a physical assumption based on a constant deceleration parameter, which allows us to properly study the universe's acceleration phase using math.

The Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi Type-IX metric of the form:

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + (b^2\sin^2 y + a^2\cos^2 y)dz^2 - 2a^2\cos y\, dxdz \tag{1}$$

where the scale factors a and b are functions of cosmic time t only.

Hoyle and Narlikar modified the Einstein field equations by adding the C-field tensor as:

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi G\left[T_{i(m)}^j + T_{i(c)}^j\right] \tag{2}$$

We define the matter field's energy-momentum tensor as $T_{i(m)}^j$ taken for a Modified Chaplygin Gas (MCG) with the equation of state:

$$p = A\rho - \frac{B}{\rho^\alpha} \tag{3}$$

where $0 \le \alpha \le 1$, and A, B are positive constants. The C-field energy-momentum tensor is given by $T_{i(c)}^j = -f\left(C_iC^j - \frac{1}{2}g_i^jC_kC^k\right)$ where $f > 0$. Assuming $C(x,t) = C(t)$, the Einstein field equations (2) for the metric (1) lead to:

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{1}{4}\frac{a^2}{b^4} = 8\pi G\left(\rho - \frac{1}{2}f\dot{C}^2\right) \tag{4}$$

$$2\frac{\dot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{3}{4}\frac{a^2}{b^4} = 8\pi G\left(-p + \frac{1}{2}f\dot{C}^2\right) \tag{5}$$

$$\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{1}{4}\frac{a^2}{b^4} = 8\pi\, G\left(-p + \frac{1}{2}f\dot{C}^2\right) \tag{6}$$

3. Solution of Field Equations

To get a definite answer, we use the C-field source equation which implies a constant rate of creation:

$$\dot{C} = 1 \tag{7}$$

Subtracting Eq. (6) from Eq. (5) eliminates the pressure term:

$$\frac{\dot{b}}{b} - \frac{\dot{a}}{a} + \frac{\dot{b}^2}{b^2} - \frac{\dot{a}\dot{b}}{ab} + \frac{1}{b^2} - \frac{a^2}{b^4} = 0 \tag{8}$$

Assuming the shear scalar σ is proportional to the expansion scalar θ, we obtain the condition:

$$a = b^n \tag{9}$$

where n is a constant ($n \ne 1$). We also assume a constant deceleration parameter q:

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \alpha \tag{10}$$

where $R = (ab^2)^{1/3}$ is the average scale factor. Integrating (10) yields the law of expansion:

$$R(t) = (mt + c_1)^{\frac{1}{1+\alpha}} \tag{11}$$

By using equations (9) and (11), we find the individual metric potentials :

$$b(t) = (mt + c_1)^{\frac{3}{(n+2)(1+\alpha)}} = T^k \tag{12}$$

$$a(t) = (mt + c_1)^{\frac{3n}{(n+2)(1+\alpha)}} = T^{nk} \tag{13}$$

where $T = mt + c_1$ and $k = \frac{3}{(n+2)(1+\alpha)}$.

Physical and Geometrical Features

Using the solutions (12) and (13) in Eq. (4), and using $\dot{C} = 1$, the matter density ρ is obtained as:

$$8\pi G\rho = \frac{m^2k^2(2n+1)}{T^2} + \frac{1}{T^{2k}} - \frac{1}{4}T^{2k(n-2)} + 4\pi Gf \tag{14}$$

Similarly, Using Eq. (5), the pressure p is calculated as:

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$$8\pi Gp = 4\pi Gf - \left[\frac{m^2k(3k-2)}{T^2} + T^{-2k} - \frac{3}{4}T^{2k(n-2)}\right] \quad (15)$$

The scalar of expansion (θ) and the shear scalar (σ) are calculated as:

$$\theta = \frac{mk(n+2)}{T} \quad (16)$$

$$\sigma = \frac{mk(n-1)}{\sqrt{3}T} \quad (17)$$

The spatial volume V is given by:

$$V = T^{\frac{3}{1+\alpha}} \quad (18)$$

The ratio of shear to expansion shows how anisotropic the system is:

$$\lim_{T\to\infty} \frac{\sigma}{\theta} = \frac{n-1}{\sqrt{3}(n+2)} \neq 0 \quad (19)$$

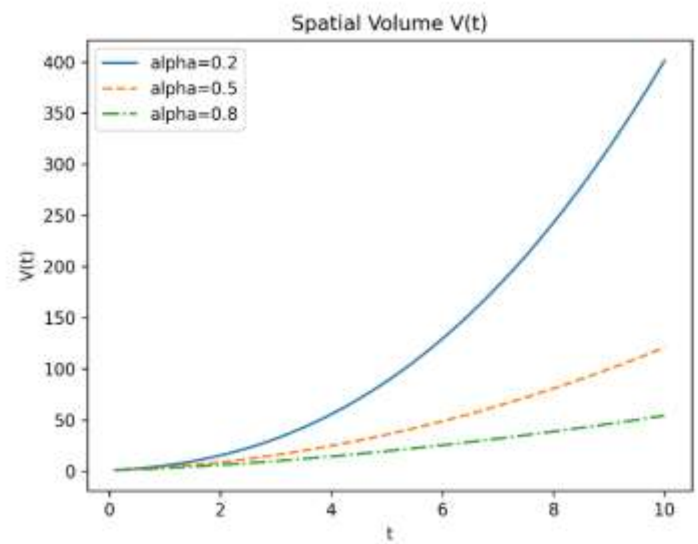
The deceleration parameter q tells us whether the expansion is speeding up or slowing down:

$$q = \alpha \quad (20)$$

For acceleration, we require $-1 < \alpha < 0$.

Discussion of Results

1. **Physical behavior of the model**



The way the space volume (V) changes over time (t) is shown in Figure 1. At the very beginning, when time is zero, the volume is very small $t = 0$, showing the start of the universe as in the Big Bang. As time goes on, the universe quickly and steadily gets bigger, and it keeps growing without end as time goes to infinity $t \to \infty$.

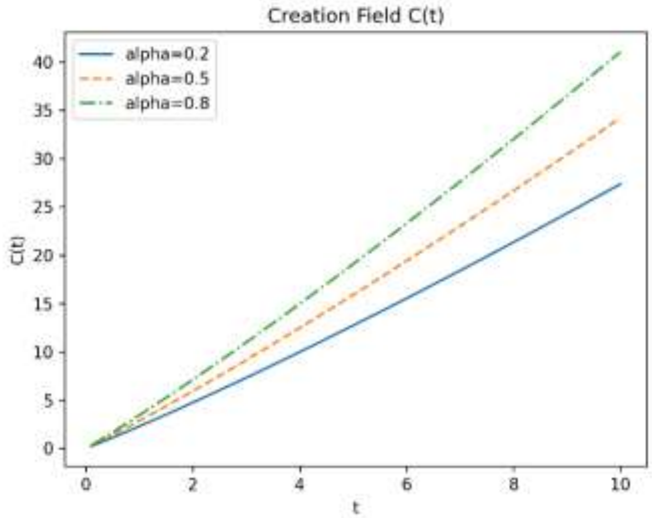
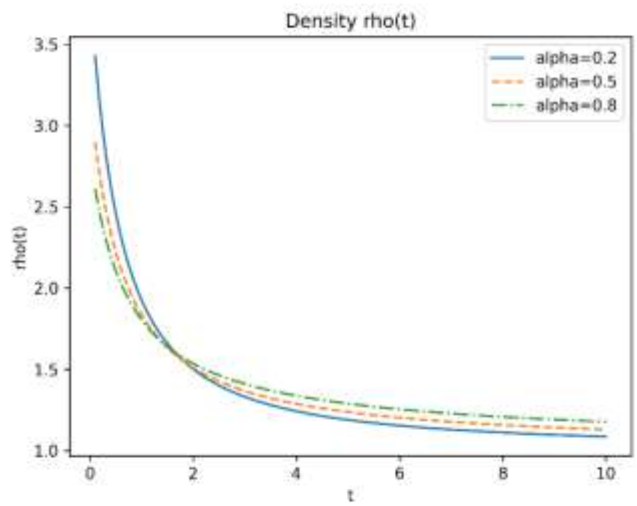


Figure 2 shows how the creation field ($C(t)$) changes over time. The graph clearly shows that the field builds up over time, starting from a limited beginning and increasing without limits. This pattern matches the basic idea of the steady-state theory, where the C-field works against the spreading out of the universe by creating new matter.



In Figure 3, we show how the density of matter (ρ) changes over time. Normally, in standard cosmological models, the energy density is expected to drop to zero ($\rho \to 0$) as the universe expands. However, our model shows something different. The density decreases slowly over time but eventually reaches a steady, positive value ($\rho \to f/2$) in the far future. This prevents the universe from ending in a future vacuum singularity.

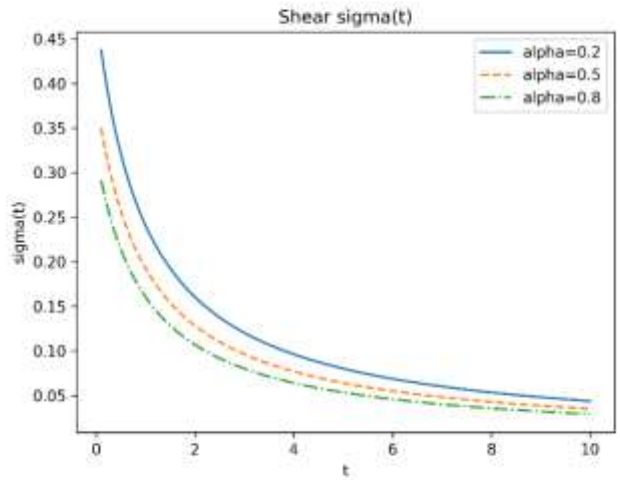


Figure 4 shows how the shear scalar (σ). changes over time. At the very beginning, when the universe starts, the shear is extremely large, but as time goes on, it gets smaller. However, the ratio of shear to expansion (σ/θ) ends up approaching a fixed, non-zero value, as long as the exponent isn't equal to one. This mathematical result shows that the universe has kept some level of uneven shape or direction throughout all of its history.

Conclusion

In this study, a Bianchi type-IX cosmological model is examined with the influence of Modified Chaplygin Gas and a Creation field, along with a constant deceleration parameter. For the range $-1 < q < 0$, the model describes an accelerating universe. At the start of the universe, it begins with infinite density, which decreases to a constant value because of the effect of the C-field ($f/2$). At the end of the universe's timeline, a "void universe" singularity does not happen in the future. Additionally, the model stays anisotropic even in the later stages, which is different from standard FRW cosmologies.

Acknowledgment
The authors express their sincere gratitude to the Department of Mathematics, Dr. Annasaheb G. D. Bendale Mahila Mahavidyalaya, Jalgaon, Maharashtra, for providing the necessary academic environment and support to carry out this research work.

Financial support and sponsorship
Nil.

Conflicts of interest
The authors have no relevant financial or non-financial interests to disclose.
Author Contributions All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Neha Murlidhar Mahajan. The first draft of the manuscript was written by Neha Murlidhar Mahajan and all authors commented on previous versions of the manuscript. V. R. Patil supervised the work and reviewed the final draft. All authors read and approved the final manuscript.

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